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**LETTER TO THE EDITOR**

# Space–time transformations in six-dimensional special relativity

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**Abstract.** A solution is found for the equations governing the space–time transformation in six-dimensional special relativity. The energy required to turn the time vector of a particle is calculated.

In his communications, Strnad (1980, 1981) raises some important questions regarding observation in the framework of relativity involving three time dimensions. He also raises the subject of the 'reciprocity condition' which has already been fully discussed elsewhere (Cole 1980a, 1981), and demands that the transformation between the space–time frames of two inertial observers should be given. In this letter we give a specific transformation, and also discuss the problem of how much energy is needed to turn the time vector of a particle.

The equations governing the transformation have been derived by Cole (1980a). Two inertial frames $S$ and $S'$ have spatial origins 0 and 0′. As seen in $S$, $\boldsymbol{v}$ is the velocity of 0′, $\boldsymbol{\alpha}_{0'}$ is its unit time vector and $\boldsymbol{\alpha}_0$ is the unit time vector of 0. As seen in $S'$, $\boldsymbol{v}'$ is the velocity of 0, $\boldsymbol{\alpha}'_0$ is its unit time vector and $\boldsymbol{\alpha}'_{0'}$ is the unit time vector of 0′. Take $c=1$ and let superscript T denote the transpose. Then the constant $6\times 6$ matrix $\Lambda$ linking the space–time coordinates of $S$ and $S'$ is of the form

$$\Lambda = \begin{pmatrix} \mathbf{A} & \mathbf{P} \\ \mathbf{Q} & \mathbf{R} \end{pmatrix}$$

where $\mathbf{A}$, $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ are constant $3\times 3$ matrices such that (in the subluminal case)

$$\begin{aligned}
&\mathbf{A}\mathbf{A}^{\mathrm{T}} - \mathbf{P}\mathbf{P}^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}}\mathbf{A} - \mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{I} \\
&\mathbf{R}\mathbf{R}^{\mathrm{T}} - \mathbf{Q}\mathbf{Q}^{\mathrm{T}} = \mathbf{R}^{\mathrm{T}}\mathbf{R} - \mathbf{P}^{\mathrm{T}}\mathbf{P} = \mathbf{I} \\
&\mathbf{A}\mathbf{Q}^{\mathrm{T}} - \mathbf{P}\mathbf{R}^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}}\mathbf{P} - \mathbf{Q}^{\mathrm{T}}\mathbf{R} = \mathbf{0} \\
&\mathbf{A}\boldsymbol{v} + \mathbf{P}\boldsymbol{\alpha}_{0'} = \mathbf{A}^{\mathrm{T}}\boldsymbol{v}' - \mathbf{Q}^{\mathrm{T}}\boldsymbol{\alpha}'_0 = \mathbf{0} \\
&\boldsymbol{\alpha}'_{0'} - \gamma(\mathbf{Q}\boldsymbol{v} + \mathbf{R}\boldsymbol{\alpha}_{0'}) = \boldsymbol{\alpha}_0 - \gamma'(-\mathbf{P}^{\mathrm{T}}\boldsymbol{v}' + \mathbf{R}^{\mathrm{T}}\boldsymbol{\alpha}'_0) = \mathbf{0} \\
&\gamma\boldsymbol{v} + \mathbf{Q}^{\mathrm{T}}\boldsymbol{\alpha}'_{0'} = \gamma'\boldsymbol{v}' - \mathbf{P}\boldsymbol{\alpha}_0 = \mathbf{0} \\
&\gamma\boldsymbol{\alpha}_{0'} - \mathbf{R}^{\mathrm{T}}\boldsymbol{\alpha}'_{0'} = \gamma'\boldsymbol{\alpha}'_0 - \mathbf{R}\boldsymbol{\alpha}_0 = \mathbf{0} \\
&\gamma\boldsymbol{\alpha}_{0'}\cdot\boldsymbol{\alpha}_0 = \gamma'\boldsymbol{\alpha}'_{0'}\cdot\boldsymbol{\alpha}'_0
\end{aligned} \tag{1}$$

where $\gamma = (1-v^2)^{-1/2}$ and $\gamma' = (1-v'^2)^{-1/2}$.

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Equation (1) replaces the 'reciprocity condition' $\boldsymbol{v}' = -\boldsymbol{v}$ which Strnad wishes to impose on the system. In the standard four-dimensional theory, this condition may be derived in the form $v'^2 = v^2$, but its imposition in the full six-dimensional theory is too restricting as an assumption because it cannot be assumed that the scalar products in (1) are equal. Only in the special case in which they are equal can it be deduced that $v'^2 = v^2$.

Strnad remarks that the above equations are not sufficient to determine $\mathbf{A}$, $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ uniquely. This is correct for $\mathbf{A}$, $\mathbf{P}$ and $\mathbf{Q}$ (this non-uniqueness also holds for $\mathbf{A}$ in the standard four-dimensional theory) because the space axes of $S$ and $S'$ may suffer any rotations about the directions of $\boldsymbol{v}$ and $\boldsymbol{v}'$ respectively to leave the above relations unchanged. In fact, let $\mathbf{U}$ and $\mathbf{U}'$ be any orthogonal matrices such that $\mathbf{U}\boldsymbol{v} = \boldsymbol{v}$ and $\mathbf{U}'\boldsymbol{v}' = \boldsymbol{v}'$, and let $\mathbf{A}$, $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ be solutions of the above equations. Then it is easily verified that the quantities $\mathbf{A}_1 \equiv \mathbf{U}'\mathbf{A}\mathbf{U}^{\mathrm{T}}$, $\mathbf{P}_1 \equiv \mathbf{U}'\mathbf{P}$ and $\mathbf{Q}_1 \equiv \mathbf{Q}\mathbf{U}^{\mathrm{T}}$ also satisfy the same equations.

In order to solve the equations, one must specify the orientations of the space–time axes. The solution may be simplified by choosing the axes in convenient directions, just as one is able to do in the four-dimensional case by taking frames in standard configuration. The general transformation may then be obtained by suitable rotations of the space and time axes. The axes may be taken such that the given vectors are $\boldsymbol{v} = (v, 0, 0)^{\mathrm{T}}$, $\boldsymbol{v}' = (-v', 0, 0)^{\mathrm{T}}$, $\boldsymbol{\alpha}_0 = (1, 0, 0)^{\mathrm{T}}$, $\boldsymbol{\alpha}_{0'} = (\cos\theta, \sin\theta, 0)^{\mathrm{T}}$, $\boldsymbol{\alpha}'_{0'} = (1, 0, 0)^{\mathrm{T}}$ and $\boldsymbol{\alpha}'_0 = (\cos\theta', \sin\theta', 0)^{\mathrm{T}}$. Equation (1) has the form $\gamma\cos\theta = \gamma'\cos\theta'$ $(\equiv a)$, and it is easily verified that the above equations have solutions

$$\mathbf{A} = \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} a & \gamma\sin\theta & 0 \\ \gamma'\sin\theta' & R_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} -\gamma'v' & P_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} -\gamma v & 0 & 0 \\ Q_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where

$$A_{11}^2(1 - a^2) + 2A_{11}a\gamma\gamma'vv' + a^2 - \gamma^2\gamma'^2 = 0 \tag{2}$$

$$R_{22}^2 = A_{11}^2 + 1 - \gamma^2 - \gamma'^2 + \gamma\gamma'\cos\theta\cos\theta'$$

$$P_{12}^2 = A_{11}^2 - \gamma'^2 \qquad Q_{21}^2 = A_{11}^2 - \gamma^2.$$

For the case $a^2 \neq 1$ the root of (2) may be taken to match with the solution for $a^2 = 1$. The result is

$$\begin{aligned} A_{11} &= a(\gamma^2\gamma'^2 - 1)(2\gamma\gamma'vv')^{-1} && \text{for } a = \pm 1 \\ &= [(\gamma^2 - a^2)^{1/2}(\gamma'^2 - a^2)^{1/2}\operatorname{sgn}(avv') - a\gamma\gamma'vv'](1 - a^2)^{-1} && \text{for } a^2 \neq 1. \end{aligned}$$

This determines $R_{22}$, $P_{12}$ and $Q_{21}$. The standard configuration of the four-dimensional theory is recovered in the special case $\theta = \theta' = 0$, in which case $\gamma = \gamma' = a$.

It is certainly true that there is no direct experimental evidence that the time vectors of a number of macroscopic particles can be different. It has been suggested (Cole 1980a) that time is strongly directed on the macroscopic scale while allowing small fluctuations about this common direction for particles on the microscopic scale. In this way, phenomena which are now regarded as 'quantum effects' will be seen to be governed by this fluctuation behaviour, with Planck's constant being in some way a measure of the interaction of matter with this time-directing field. If this is the case then it should be possible to derive some of the properties of quantum mechanics by imposing some restriction on the geometry of the six-dimensional space–time.

It is possible to calculate the energy required to turn the time vector of a particle. The energy of a particle is now a vector directed along its time vector (Cole 1980a). Suppose the particle is at rest in frame $S$ with mass $m$ and time vector $\boldsymbol{\alpha}$, and it is required to change this time vector to $\boldsymbol{\beta}$ while still keeping the particle at rest in $S$. The energy needed for this change is $\boldsymbol{E} = mc^2\boldsymbol{\beta} - mc^2\boldsymbol{\alpha}$, or

$$|\boldsymbol{E}| = 2mc^2 \sin(\theta/2) \tag{3}$$

where $\theta$ is the angle between $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. Thus, for example, the energy magnitude required to turn the time vector of a macroscopic particle of mass 1 kg through 1 degree is $1.57 \times 10^{15}$ J. This energy is just not available in normal macroscopic situations to produce noticeable deviations in the time subspace. However, since the energy needed is proportional to $m$, sufficient energy should be available on the microscopic scale to produce phenomena which are now classed as quantum effects.

It would be most satisfactory if the main results of quantum mechanics could be put on a very simple basis using this theory, and that some way could be found of generating the new electromagnetic fields predicted by the theory (Cole 1980b).

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